International Journal of Applied Linguistics & English Literature
ISSN 2200-3592 (Print), ISSN 2200-3452 (Online)
Vol. 2 No. 4; July 2013

Mathematical Approaches to Cognitive Linguistics

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Received: 13-04-2013 Accepted: 24-05-2013 Published: 01-07-2013
doi:10.7575/aiac.ijalel.v.2n.4p.192 URL: http://dx.doi.org/10.7575/aiac.ijalel.v.2n.4p.192

Abstract
Cognitive linguistics, neuro-cognitive and psychological analysis of human verbal cognition present important area of multidisciplinary research. Mathematical methods and models have been introduced in number of publications with increasing attention to these theories. In this paper we have described some possible applications of mathematical methods to cognitive linguistics. Human verbal perception and verbal mapping deal with dissipative mental structures and symmetric/asymmetric relationships between objects of perception and deep (also surface) structures of language.
In that's way methods of tensor analysis are ambitious candidate to be applied to analysis of human verbal thinking and mental space.
Keywords: Mental mapping, verbal cognition, continuum hypothesis, Hilbert space, tensor transformations, vector space model, human mental lexicon, representational granularity, embodied and symbolic cognition, mental models.

1. Introduction

Cognitive linguistics, in close relation with neuro-psychological research of human verbal perception presented dynamical approach to linguistics created new framework for multidisciplinary research. Object of research in cognitive linguistics, complexity and multi-dimensionality of phenomena have required re-thinking a methodology of research by applying more powerful theories and methods of mathematics. Researchers in USA, European countries, Russia and Japan have introduced mathematical concepts and models in psychological and neuro-cognitive studies of language. Human verbal perception, phenomena of embodied and symbolic cognition, metaphor are an object of multi-disciplinary research both in linear and non-linear frameworks. This framework provides sound basis to apply powerful ideas of quantitative, mathematical methods to cognitive linguistics.

Tensor theory, Vector transformations in finite and non-finite continuum and in Hilbert space serve as a basis for modeling above named phenomena of human verbal thinking and in that's way offer new perspectives in extension of the theory of Universal grammar. Application of mathematical models in cognitive linguistics lead to development of comparatively new area of multidisciplinary which should be called as a quantum semantics, or dynamic model of Universal grammar.

Author of this paper presented some ideas in applying mathematical methods and ideas to cognitive research. These ideas have been used in research in cognitive linguistics including empirical data from typologically different languages like Mongolian, English and Russian.

1.1 Continuum (Cantor G) and Hilbert space in cognitive linguistics

In the light of cognitive research, mental states (in human brain) served as a basis for generating meanings which are producing high-order syntax structures, discourses.

In that's way applying an idea of Continuum hypothesis to modeling mental states and meanings are useful to describe human verbal thinking categories and primitives. Mental states are infinite (infinite sets) and the size of these sets, their cardinality, is infinite($/N/=\aleph_0$).

Cantor's theorem "the cardinality of the power set $\wp\ P(S)$ (set of all subsets of S) is greater than the cardinality of *S:* $/S/< /\wp(S)/$ has direct links to analysis of human mental space: power of subsets of propositions (meanings) in discourse is greater than sum of propositions (meanings) in mental discourse. But words and rules (grammatical and transformation rules) are finite sets.

In that's way concept of cardinality has an significance for comparing infinite sets of mental concepts, meanings and finite sets of vocabulary, grammatical rules and syntax transformations, bijection between these two kinds of sets.

It means than Cantor's continuum hypothesis, particularly notion of set in combination with latest results of psychological and neurolinguistics researches presented new opportunities to develop theories of generative syntax and semantics, particularly ideas of UG in new dimensions (Chomsky Ch. 1993).

Modeling mental states and meanings as an infinite sets is leading to concept of Hilbert space (Naohito Chino, 1998) which extends the methods of vector algebra from the two – dimensional Euclidean plane and three – dimensional space to spaces with any finite or infinite number of dimensions. Hilbert space as an abstract vector space served as an effective method to modeling macro (semantic) structure of mental discourse, its latent organization, mental force and coherence, cohesion.

Tensor (Vector) analysis in combination with ideas of continuum and Hilbert space of support an interpretation of storing words in a memory, their distribution and relationships capacity of Semantic memory, neurocognitive mechanism of semantic organization of vocabulary, syntax of high order mental structures.

2. Tensor analysis in cognitive linguistics

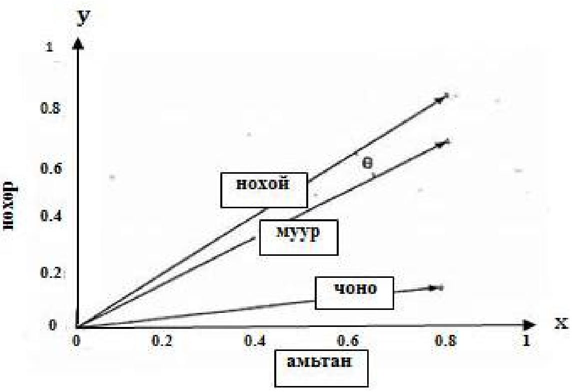
Formalization of Semantic space in terms of Hilbert space is background to apply tensor models to analysis of vocabulary and syntax structures in psychological and neurocognitive linguistics.

2.1 Measuring human mental lexicon

In n – dimensional semantic spaces standard Euclidean distance function is used for measuring the similarity between two words or concepts and the similarity is partially defined by the degree to which their features (sensory, motor, affective features such as shape, size, color, distance, location) overlap, the degree to which they share same contexts are basis to measure direction and magnitude (size, force) of relations between words in mental lexicon. Many pattern recognition techniques are based on similarity measures between objects (nearest neighbor classification, cluster analysis, multi-dimensional scaling).

The example in which two-dimensional semantic (metric) space between representatives of subsets is measured by using standard Euclidean distance function:

$$d(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2} \qquad (1)$$



нөхөр-friend,
амьтан- animal,
нохой- dog,
муур-cat,
чоно-wolf.

Fig 1. Semantic space between words

Measuring semantic space between words (concepts) is not point of analysis. Metric space (Euclidean, Minkowski, Hausdorff etc.) is important starting point for interpretation of psycho-cognitive operations as an operations of mental grammar, rules of verbal thinking. The similarity between words and concepts in Human semantic space presents interest for analysis of the phenomenon of generalization and mutual exclusivity in terms of continuum and Hilbert space. Infant extend labels on the basis of category membership (principle of categorical scope) to avoid second labels for objects (mutual exclusivity). So for infant multiple meaning is different words. In contrast to it, infant also used a principle of generalization. So in this case Euclidean distance is simplest variant of generalization and mutual exclusivity in human mental space.

Extension of the principle of generalization and mutual exclusivity to an area of classes of words or concepts is leading to ideas of fussy sets. Several authors have proposed similarity indexes for fuzzy sets can be viewed as generalizations of the classical set-theoretic similarity functions. A systematic investigation of this notion was performed by Dubois and Prade.

$$|A| = \sum_{x \in X} \mu_A(x) \qquad (2)$$

The membership function as a representation of magnitude and scalar cardinality of a fuzzy set must to serve as a basis for classification of words a syntax structures. In his well- known paper entitled "Features of Similarity" (Tversky) similarity among objects is expressed as a linear combination of the measure of their common and distinct features. Each object in domain D is represented by a set of features or attributes, and A, B and C denote the set of features associated with objects a, b, and c, respectively.

Representation of two objects that each contains its own unique features also contains common features. An important aspect of Tversky's model is that similarity depends not only on the proportion of features common to the two objects but also on their unique features.

Euler diagram presents good example to supporting an idea of description attributive properties and values in the fuzzy sets, subsets and on this basic for interpretation of degree of membership of words and concepts in human mental lexicon. /Khan H. 2012/

Example:

Max (meat) : Set of all products animals that are meat.
Cүү (milk) : Set of all animals that are milk.
Шинэ (new) : Set of all fresh products.

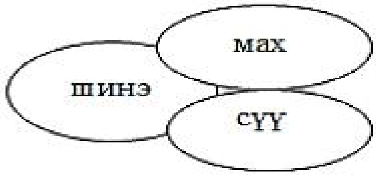


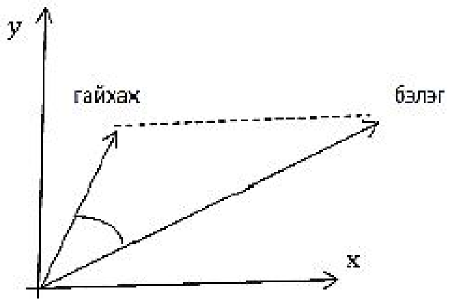
Fig 2. Intersection of members in mental subset

Notion of scalar cardinality of fuzzy sets in supporting an analysis of Human mental lexicon and syntax in combination with an idea of metric space (Human Semantic space).

2.2 Modeling tectonics of syntax structures

The distances between words are also structural characteristic of sentences. More easy formula is the cosine of two vectors by using Euclidean dot product formula.

$$similarity = \cos(\theta) = \frac{A \bullet B}{\|A\|\|B\|} = \frac{\sum_{i=1}^{n} A_i \times B_i}{\sqrt{\sum_{i=1}^{n} (A_i)^2} \times \sqrt{\sum_{i=1}^{n} (B_i)^2}} \quad (3)$$



Би бэлэг аваад гайхсан.
I have been exhausted with receiving this gift.
(гайхах–exhausted, бэлэг –gift)

Fig 3. Distance between words in a sentence

Analysis of word distance based on vector (tensor) models has close relation to statistical similarity measures between sentences. In this connection sentence similarity measure presented by Junsheng Zhang Yunchuan Sun, Huiling Wang and Yanging He will be developed with application of tensor ideas. (Junsheng Zhang, Yunchuan Sun, Huiling Wang, 2011). Co-occurrence of words in sentence reflects relationships between words (word categories) in N-dimensional space presenting complexity of semantic and thus human mental regions.

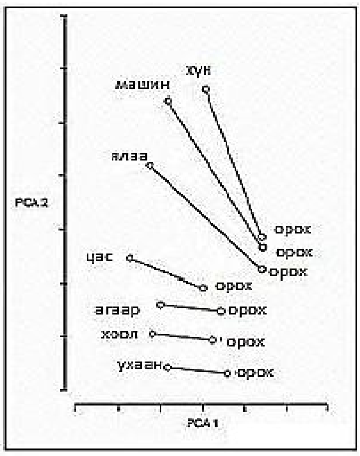


Fig 4. Co-occurrence of words in semantic (mental) space

Measuring co-occurrence of words, distance similarity between words in a sentence by using vector (tensor) models interesting application is must be applied to an analysis of tectonics of syntax structures and thus of word/ sentence recognition patterns.

One of phenomena at syntax level is a permutation and substitution which is related to combination of semantic and grammatical categories. In this connection the Levenshtein distance between two strings has some importance. Tensor-based permutation particularly special unitary group transformations, irreducible representations of *SU* (n) are oriented Researchers to rethinking a permutation in Linguistics as an act of changing the order of elements, transformations.

Eigenvalues and Eigenstates in linguistics are basis for analysis of transformations as dynamic properties of human mental states.

Angle preserving similarity transformation and rigid transformation (distance preserving), affine transformation (parallelism preserving) has taken place in syntax of different languages. So these classes of transformations must be considered as properties of mental syntax generating linguistic universals.

Transformations of syntax structures based on mental mapping and embedding depend on space-time coordinatization in human mind. According to differential geometry, distance between two events in space – time is presented depending on particular coordinatization system. Intrinsic features of an object in space – time coordinate characterize the surface independently of any particular coordinatization systems. Extrinsic features are mere artifacts of the form of representation. In our opinion intrinsic and extrinsic features in combination have caused difference in syntax structures. Interpretation of intrinsic and extrinsic features in terms of representational granularity needed to consider following neurocognitive processes of verbal thinking. Strong influence of extrinsic features on an object must originate a difference in color presentation in typologically different languages as a Mongolian, Russian, English and Chinese. Color is recognized faster than size because color is not a relative property. Head - noun referring, structure with two satellites referring to color and motion (хөөрсөн улаан бөмбөлөг, гүйж яваа өндөр хүү), computational algorithms on determinism (minimally specified expressions. over specified expressions. The big red car. The red big car), GRZ algorithms (Generation of referring expressions-“size” has great discriminative power) also present an example for analysis of human computational algorithms with vector application to in differential geometry. (Deemer, K. V. CS. 2012. 36/5 P 830).

Difference in object localization is an example of coordinatization dependent of perceptual granularity for which scalar product (scalar triple product) is basic interpretation.

Энд ширээ байна.

Өрөөнд хивс дэвсээтэй байна. (Mongolian)

Здесь стоит стол.

В комнате лежит ковер. (Russian)

There is a table.

There is a carpet in the room. (English)

Notion of representational granularity is very important for testing symbol interdependency hypothesis because language comprehension is both embodied and symbolic. Human algorithm on sequence fixation also must be interpreted in terms of tensor (vector) models. Specific object of tensor analysis are differences in event mapping:

SOV – Би ном авсан. (Mongolian)

SVO – The man opened the letter. (English)

SVO - Парень открыл дверь (Russian)

Tensor (Vector) in terms of their transformations presents effective way for describing mapping of the event components to linguistic structure, particularly mapping in argument structure an object, motion paths, its manner, source and goal of motion. In tensor-based mapping trajectory of motion, its speed manner are choremes. According to Klippel A., conceptual primitives combine movement choremes into chunks. Chunks (CMPs) are the basis for a conceptual grammar of movement patterns. (Klippel, A. Topics in CS.2011.3/4. P722).

In tensor analysis choremes should be included as a proper of direction and magnitude. Chunks are considered as a vector multiplication in this modeling. Differences in coordinatization are surface illustration of change in vector direction and magnitude in mental mapping. For example in Mongolian and English languages source /agents should be more prominent than goal/ patients for vector transformation.

Giving | Багш хүүд ном өгөв.
taking | Хүү багшаас ном авав.

Throwing | The girl threw the ball to the boy.
catching | The girl caught the ball from the boy.

Source and goal paths in conceptually different domains is an object of such transformations:

Дорж машинаас онгоцонд суув.

Дорж ажилчинаас эзэн суув.

Similar examples in English.

Brian went from the car to the store.

Brian went from sad to happy.

Multiple frames of reference in egocentric and ethnocentric coordinates of mental imagery underlying different structures also present interest in tensor-based modeling of human perceptual mapping. Change of one coordinate frame towards the other in the opposite direction of the representation is specific example of perceptual mapping.

Хүүхэд аяга хагалчихлаа. - I dropped the glass. | Causative relations

Аяга хагарчихлаа. - The glass fell.

Бөмбөг хаалганд оржээ. | Incorporating location into the goal

Пуужинг далай дээгүүр хөөргөв.

Structural priming, overlapping of primes and targets in event structure is basis for symmetry and asymmetry in human perceptual mapping.

Бат машинаа хашаанд оруулав. Bat drove the car into the garage.

Бат хашаанд машинаа оруулав. Bat drove into the garage the car.

Бат хүүд номоо өгөв. (Bat gave – the son – a book

Бат номоо хүүд өгөв. Bat gave – a book – to the son).

Asymmetry in mapping, correlation between attention window and sentence structure also have importance in the light of tensor transformations. Tensor models present effective way to interpret a complexity of relations between components of syntax structures. Complex sentences (object – extracted relative clauses-ORC, subject–extracted relative clauses-SRC, left–branching and right–branching constructions), retrieval difficulty also should be interpreted by using tensors.

Number of words that intervene between the relevant syntactic dependents, number of referents intervening between the two elements of a long-distance dependency, decay-based metric and decay as a function of working memory is good example to apply tensors to analysis of human mental mapping and lexicon.

Хүү зам буруу заасан алдаагаа ойлгожээ.

Зам буруу заасан алдаагаа хүү ойлгожээ.

Tensor-based modeling of syntax supports to an idea that topological transformations (shortening, lengthening, and moving of temporal intervals) are a basis for transformations in mental syntax.

In that's way tensor is effective application to analysis of connectionist links of mental representations, analogical mapping, semantic transformations.

In addition to tensors, for description of central and periphery structures in syntax must be applied intermediate value theorem (Bolzano) for more complex analysis of such structures and other structures related to prototype-plus-distortion phenomenon (central prototype and radial prototype categories).

Prototype: Эгч талх зүсэв. The sister cuts the bread.

Radial Categories	Force ≈ voluntary agent: Цас хаалга даржээ. - Салхинд хаалга хаагджээ. The wind closed the door. Instrument: Чулуу цонх хагалжээ. The hammer broke the window. Neutral: Дэмбээ ууланд авирав. Peter climbed the mountain /O not affected by S/

Tensor which generalizes the concepts of tensor in a way that is independent of any chosen frame of reference has an importance for analysis such structures at deep and surface levels. Interpretation of transformation SOV- OVS in three dimensional space is an effective instrument describe a tectonics of typologically different syntax structures.

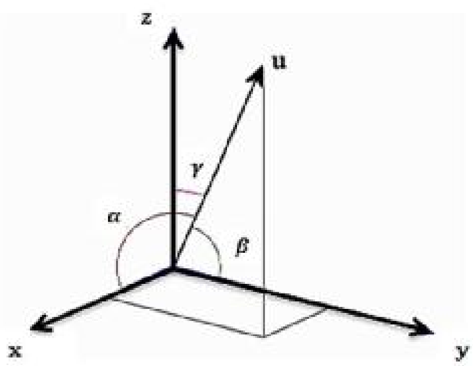


Fig 5. Vector space of syntax structures

α is the angle between **u** and the *x*-axis (in dark red),
β is the angle between **u** and the *y*-axis (in green) and
γ is the angle between **u** and the *z*-axis (in pink),

Unit vector based on direction cosines is **u** = cos α **i** + cos β **j** + cos γ **k** must describe a difference in transformations (SOV-OVS).
The 3-dimensional version of the Pythagorean Theorem can be used for finding the distance between points in the 3-dimensional coordinate system

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \quad (4)$$

Example: Би чоно харсан. – Чоно надад харагдсан.
My father gave me this watch. – I was given this watch by my father.

In three dimensional space S and P (V) are carriers of nucleus force and composing nuclei (nucleus) of human basic unit of thought (as a proposition).
Tensor-based analysis is applied to mapping sequence of words to semantic roles mapping sentence structures by using example-based or category-based scheme (transitive→ non-transitive, active → passive structures) as an indicator of human cognitive development.
Differences in verbal presentation of events having same structure (intrinsic features are same) and objects with similar localization in time – space dimensions mean that mapping an event or object in the brain is differing, varying. This factor served as one of the origins of differences in pragmatic transformations and interpretations.
Good examples of dependency of human perceptual mapping on coordinatization are metaphors.
Ego-moving metaphor: Бид хотод дөхөж байна. (We are approaching the city).
Time-moving metaphor: Цагаан сар дөхөж байна. New Year (traditional) is approaching.
Object-moving metaphor: Хот дөхөж байна. (The city is approaching).
Modality specific sensorimotor encodings as a base domain from which metaphors are drawn have related to basic primitives for tensor (vector) transformations. This our suggestion is closely connected to results of experiment what projections and mappings in human brain are realized by bodily-grounded experience as visual perception, spatial experience, thermic experience and so on. This fact is oriented us to bodily-grounded description of human associative mechanism which is very significant for building metaphor, non-linear verbal thinking, embodied and symbolic cognition.
Tensor transformations, covariance and contravariance preserving the space-time interval between any two events are most powerful basis to analyse mentally-based verbal transformations. Dependency/independency of the coordinatization system has close relation to a typology of languages and transformations of syntax structures present examples of linear and non-linear transformations on linguistics. It means that in linguistics powerful ideas like Lorenz transformation has methodological importance for re-vision of linguistics relativity, thus for interpretation of linguistic relativity hypothesis of Sapir-Whorf.
2.3 Modeling discourse structure
Discourse building and comprehension is an embedding based on mental models. In that's way discourse presents interesting object for interpretation by applying quantitative models. Discourse as a multidimensional distribution of latent topics, topics - as a multidimensional distribution of words are be described in the form of direction and magnitude in these models. Metric or ordering information are basic features of primitives to be put in such models (Klippel, A. Topics in CS.2011.3/4.P726).
Nonstandard texts based mostly on latent propositions and anomalies in space-time causative relations, constructions keeping conflict information mobilizing right hemisphere present a specific object of tensor (vector) transformations. Discourse structure and human memory limitations, sublinear increase in the cumulative discourse informativity (as the sum of all the information provided by the words in that discourse) also object for tensor analysis. Cue effectiveness decay hypothesis (the effectiveness of cues decreases as a power law with increasing distance) is also must be tested in tensor model.

Good basis for modeling a discourse is a notion of conceptual space. Gärdenfors proposes representing concepts as regions in a conceptual space, whose quality dimensions correspond to interpretable features which are types of sensory perception. According to Gärdenfors, properties (color, temperature) are properties that occupy convex regions in conceptual space, proposing that all properties used in human cognition are natural. (Gärdenfors. 2000)

In psychology, vector based models of human concept representation are used for interpretation of mental categorization. Since many experiments on human concept representation have been performed by using verbal cues, these models represent aspects of word meaning, possibly along with other types of knowledge. Nosofsky's Generalized Context Model (GCM) is an example of it (Nosofsky, R. 1990).

In tensor-based discourse modeling researcher consider that each dimension of the Hilbert space *H* corresponds to a word in the vocabulary of a corpus of documents. And in *n*-dimensional (real valued) Hilbert space H, the inner product is represented by the Euclidean scalar product. The Easiest way to apply tensor (vector) analysis is to compare different discourses by using graph-based models. In this connection Euler's theorem (fundamental theorem in graph theory): V-E+F=2 will be tested in linguistic research. Comparing the word vectors is one way to derive meaning from the semantic space. An alternative is to compare subspaces of documents, or sets of documents. Simplest solution is to consider the distance between the representation of the same word vector in the semantic space, using for example the Euclidian distance, Minkowski distance. In this case a span of text is a vector in V-dimensional space and both document (doc or text) and queries can be represented as vectors. Effective way to compute similarity between two vectors is the cosine of the angle between these vectors.

$$\frac{\sum_{i=1}^{V} x_i \times y_i}{\sqrt{\sum_{i=1}^{V} x_i^2} \times \sqrt{\sum_{i=1}^{V} y_i^2}} \quad (5)$$

There is important measuring inner product and TF IDF /term frequency–inverse document frequency/, queries and documents are represented as vectors of TF IDF weights.

An example of Vector space representation of discourse (document) in Mongolian language:

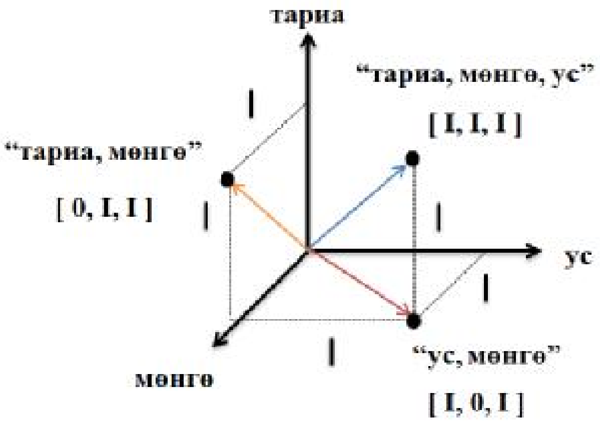


Fig 6. Similarity measure-inner product

	ус	тариа	мөнгө
doc_1	1	1	1
doc 2	1	0	1
doc_3	0	1	1

Table 1. Vector space similarity

The vector space model ranks documents (texts) based on the cosine angle between the query and each document.

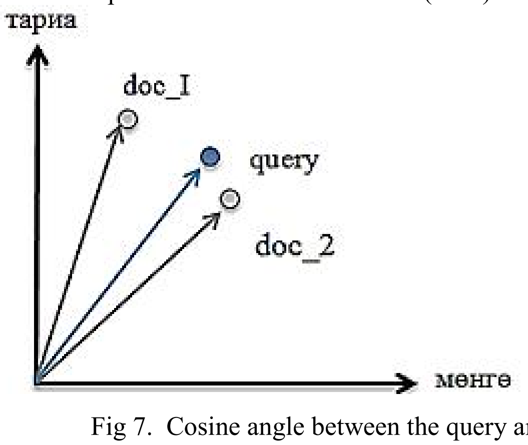


Fig 7. Cosine angle between the query and documents

From a cognitive perspective, in the vector space model of discourse the query and TF IDF have an importance for a representation of prototypes as $n-$ dimensional regions in human mental space. Discourse having N-dimensional semantic space and multiple (or non-linear) links between subspaces also should be described in terms of Hausdorff metric and Chordal distance, which expresses the similarity (driven by the subspace distance) between subspaces. Tensor analysis in Hilbert space is one of effective ways to analyse implicitness of some substructures or propositions of discourse structure, and must be applied to interpretation of music and painting, to analysis of differences and similarities in syntax of verbal and musical production as a specific types of Human discourse. Vector-based discourse analysis served as a framework to formalize human conceptual spaces as vector spaces.

3. Discussion & Conclusion

In this paper we have discussed possible applications of some mathematical methods to research in psychological and cognitive linguistics. Phenomena of verbal perception and mental mapping, symmetry/asymmetry of mental and linguistics primitives present an attractive alternative to apply tensor vector models to cognitive linguistics. Human mental lexicon, structural priming in mental and surface syntax, multidimensional structure of discourse presented in mathematical models must have significant theoretical and practical implications. These methods and models are leading to more precise description of differences and similarities in typologically different languages extending standard framework of typological linguistics. Hilbert space as an analogy of human mental space should serve as a basis for analysis of human mental primitives. Tensor-based description of human algorithms in relation to syntax and discourse structures present one of models for dynamic grammar of Universal theory of language. Synthesis of ideas of representational granularity in human mental mapping and computational algorithms with covariance and contra variance is effective way to interpret interrelations between human perceptual mapping and tectonics of syntax and discourse, linearity and non-linearity in these relations. Tensor-based analysis of discourse structure as a conceptual space in the framework of Hilbert space could find interesting application in modeling Human mental space, multidimensional verbal structures.

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